9 TREE-STRUCTURED INDEXING

I think that I shall never see
A billboard lovely as a tree.
Perhaps unless the billboards fall
I'll never see a tree at all.

—Ogden Nash, *Song of the Open Road*

We now consider two index data structures, called ISAM and B+ trees, based on tree organizations. These structures provide efficient support for range searches, including sorted file scans as a special case. Unlike sorted files, these index structures support efficient insertion and deletion. They also provide support for equality selections, although they are not as efficient in this case as hash-based indexes, which are discussed in Chapter 10.

An ISAM[1] tree is a static index structure that is effective when the file is not frequently updated, but it is unsuitable for files that grow and shrink a lot. We discuss ISAM in Section 9.1. The B+ tree is a dynamic structure that adjusts to changes in the file gracefully. It is the most widely used index structure because it adjusts well to changes and supports both equality and range queries. We introduce B+ trees in Section 9.2. We cover B+ trees in detail in the remaining sections. Section 9.3 describes the format of a tree node. Section 9.4 considers how to search for records by using a B+ tree index. Section 9.5 presents the algorithm for inserting records into a B+ tree, and Section 9.6 presents the deletion algorithm. Section 9.7 discusses how duplicates are handled. We conclude with a discussion of some practical issues concerning B+ trees in Section 9.8.

Notation: In the ISAM and B+ tree structures, leaf pages contain *data entries*, according to the terminology introduced in Chapter 8. For convenience, we will denote a data entry with search key value k as $k*$. Non-leaf pages contain **index entries** of the form $\langle$*search key value, page id*$\rangle$ and are used to direct the search for a desired data entry (which is stored in some leaf). We will often simply use *entry* where the context makes the nature of the entry (index or data) clear.

[1]ISAM stands for Indexed Sequential Access Method.

9.1 INDEXED SEQUENTIAL ACCESS METHOD (ISAM)

To understand the motivation for the ISAM technique, it is useful to begin with a simple sorted file. Consider a file of Students records sorted by *gpa*. To answer a range selection such as "Find all students with a gpa higher than 3.0," we must identify the first such student by doing a binary search of the file and then scan the file from that point on. If the file is large, the initial binary search can be quite expensive; can we improve upon this method?

One idea is to create a second file with one record per page in the original (data) file, of the form ⟨*first key on page, pointer to page*⟩, again sorted by the key attribute (which is *gpa* in our example). The format of a page in the second *index* file is illustrated in Figure 9.1.

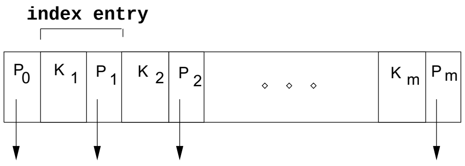


Figure 9.1 Format of an Index Page

We refer to pairs of the form ⟨*key, pointer*⟩ as *entries*. Notice that each index page contains one pointer more than the number of keys—each key serves as a *separator* for the contents of the pages pointed to by the pointers to its left and right. This structure is illustrated in Figure 9.2.

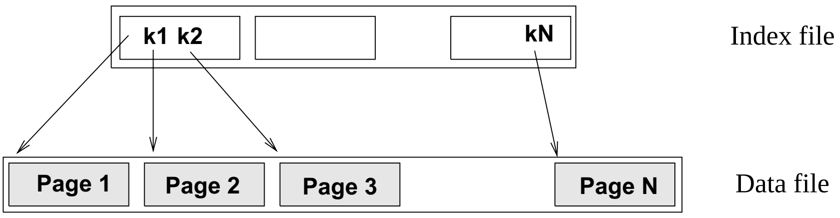


Figure 9.2 One-Level Index Structure

We can do a binary search of the index file to identify the page containing the first key (*gpa*) value that satisfies the range selection (in our example, the first student with *gpa* over 3.0) and follow the pointer to the page containing the first data record with that key value. We can then scan the data file sequentially from that point on to retrieve other qualifying records. This example uses the index to find the first data page containing a Students record with *gpa* greater than 3.0, and the data file is scanned from that point on to retrieve other such Students records.

Because the size of an entry in the index file (key value and page id) is likely to be much smaller than the size of a page, and only one such entry exists per page of the data file, the index file is likely to be much smaller than the data file; thus, a binary search of the index file is much faster than a binary search of the data file. However, a binary search of the index file could still be fairly expensive, and the index file is typically still large enough to make inserts and deletes expensive.

The potential large size of the index file motivates the ISAM idea: Why not apply the previous step of building an auxiliary file on the *index* file and so on recursively until the final auxiliary file fits on one page? This repeated construction of a one-level index leads to a tree structure that is illustrated in Figure 9.3. The data entries of the ISAM index are in the leaf pages of the tree and additional *overflow* pages that are chained to some leaf page. In addition, some systems carefully organize the layout of pages so that page boundaries correspond closely to the physical characteristics of the underlying storage device. The ISAM structure is completely static (except for the overflow pages, of which it is hoped, there will be few) and facilitates such low-level optimizations.

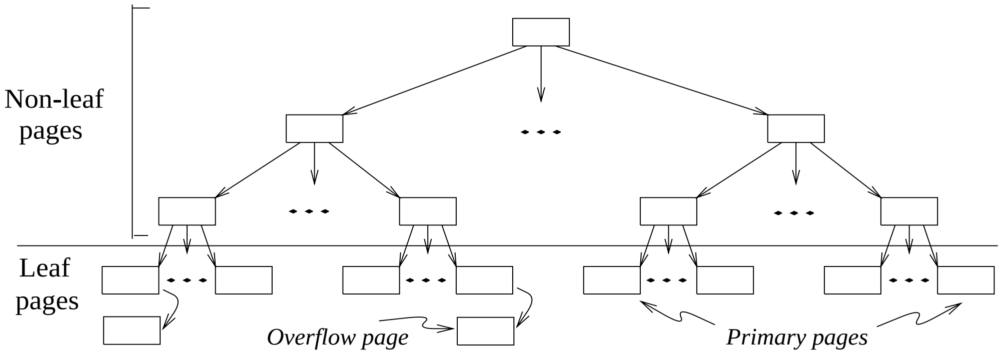


Figure 9.3 ISAM Index Structure

Each tree node is a disk page, and all the data resides in the leaf pages. This corresponds to an index that uses Alternative (1) for data entries, in terms of the alternatives described in Chapter 8; we can create an index with Alternative (2) by storing the data records in a separate file and storing ⟨*key, rid*⟩ pairs in the leaf pages of the ISAM index. When the file is created, all leaf pages are allocated sequentially and sorted on the search key value. (If Alternatives (2) or (3) are used, the data records are created and sorted before allocating the leaf pages of the ISAM index.) The non-leaf level pages are then allocated. If there are several inserts to the file subsequently, so that more entries are inserted into a leaf than will fit onto a single page, additional pages are needed because the index structure is static. These additional pages are allocated from an overflow area. The allocation of pages is illustrated in Figure 9.4.

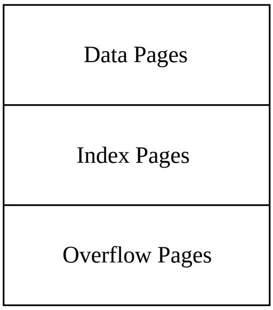


Figure 9.4 Page Allocation in ISAM

The basic operations of insertion, deletion, and search are all quite straightforward. For an equality selection search, we start at the root node and determine which subtree to search by comparing the value in the search field of the given record with the key values in the node. (The search algorithm is identical to that for a B+ tree; we present this algorithm in more detail later.) For a range query, the starting point in the data (or leaf) level is determined similarly, and data pages are then retrieved sequentially. For inserts and deletes, the appropriate page is determined as for a search, and the record is inserted or deleted with overflow pages added if necessary.

The following example illustrates the ISAM index structure. Consider the tree shown in Figure 9.5. All searches begin at the root. For example, to locate a record with the key value 27, we start at the root and follow the left pointer, since $27 < 40$. We then follow the middle pointer, since $20 <= 27 < 33$. For a range search, we find the first qualifying data entry as for an equality selection and then retrieve primary leaf pages sequentially (also retrieving overflow pages as needed by following pointers from the primary pages). The primary leaf pages are assumed to be allocated sequentially—this assumption is reasonable because the number of such pages is known when the tree is created and does not change subsequently under inserts and deletes—and so no 'next leaf page' pointers are needed.

We assume that each leaf page can contain two entries. If we now insert a record with key value 23, the entry 23* belongs in the second data page, which already contains 20* and 27* and has no more space. We deal with this situation by adding an *overflow* page and putting 23* in the overflow page. Chains of overflow pages can easily develop. For instance, inserting 48*, 41*, and 42* leads to an overflow chain of two pages. The tree of Figure 9.5 with all these insertions is shown in Figure 9.6.

The deletion of an entry $k*$ is handled by simply removing the entry. If this entry is on an overflow page and the overflow page becomes empty, the page can be removed. If the entry is on a primary page and deletion makes the primary page empty, the simplest approach is to simply leave the empty primary page as it is; it serves as a

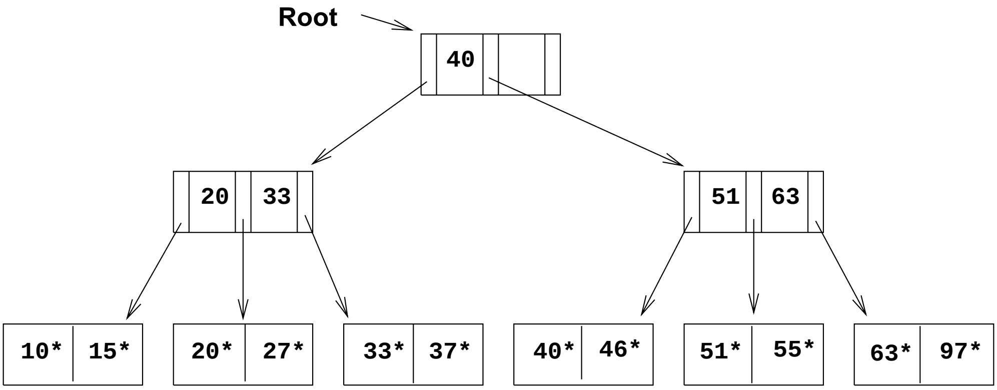


Figure 9.5 Sample ISAM Tree

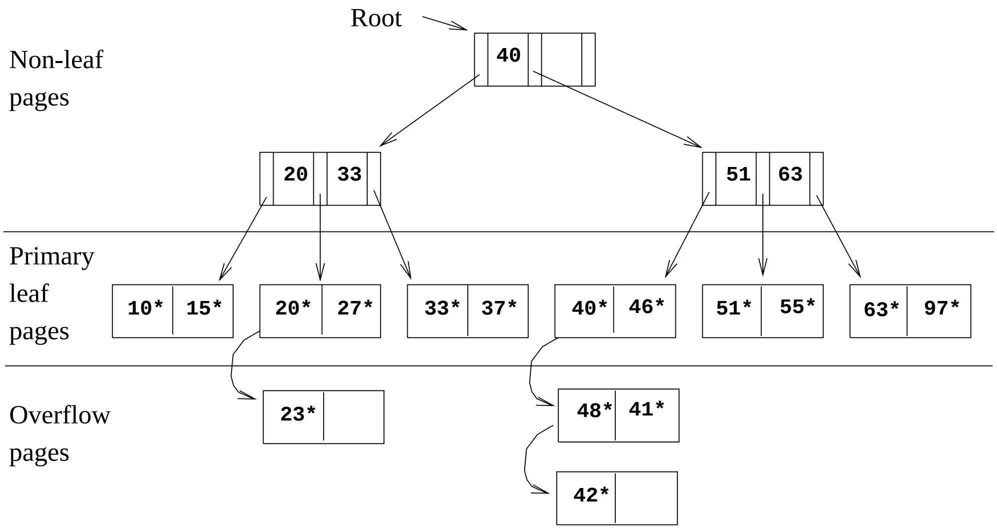


Figure 9.6 ISAM Tree after Inserts

placeholder for future insertions (and possibly non-empty overflow pages, because we do not move records from the overflow pages to the primary page when deletions on the primary page create space). Thus, the number of primary leaf pages is fixed at file creation time. Notice that deleting entries could lead to a situation in which key values that appear in the index levels do not appear in the leaves! Since index levels are used only to direct a search to the correct leaf page, this situation is not a problem. The tree of Figure 9.6 is shown in Figure 9.7 after deletion of the entries 42*, 51*, and 97*. Note that after deleting 51*, the key value 51 continues to appear in the index level. A subsequent search for 51* would go to the correct leaf page and determine that the entry is not in the tree.

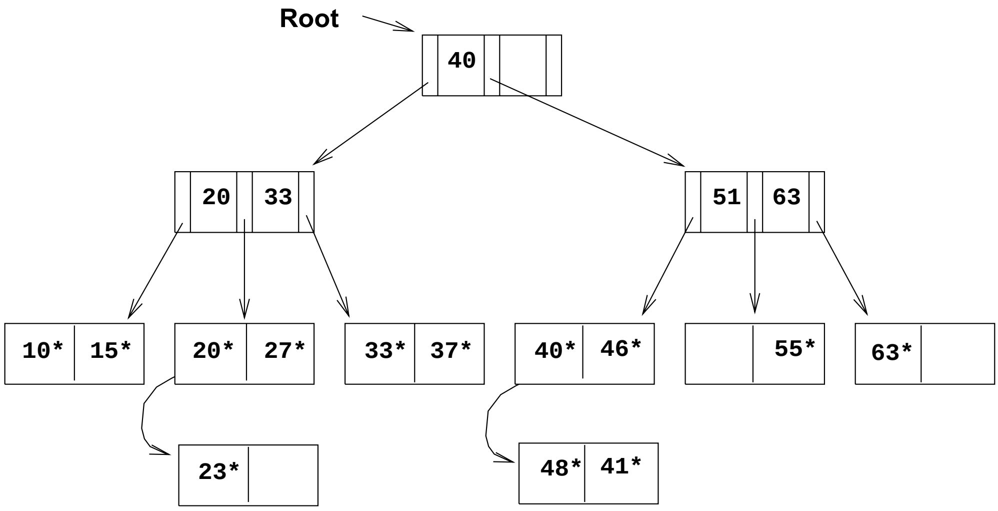


Figure 9.7 ISAM Tree after Deletes

The non-leaf pages direct a search to the correct leaf page. The number of disk I/Os is equal to the number of levels of the tree and is equal to $log_F N$, where N is the number of primary leaf pages and the **fan-out** F is the number of children per index page. This number is considerably less than the number of disk I/Os for binary search, which is $log_2 N$; in fact, it is reduced further by pinning the root page in memory. The cost of access via a one-level index is $log_2(N/F)$. If we consider a file with 1,000,000 records, 10 records per leaf page, and 100 entries per index page, the cost (in page I/Os) of a file scan is 100,000, a binary search of the sorted data file is 17, a binary search of a one-level index is 10, and the ISAM file (assuming no overflow) is 3.

Note that once the ISAM file is created, inserts and deletes affect only the contents of leaf pages. A consequence of this design is that long overflow chains could develop if a number of inserts are made to the same leaf. These chains can significantly affect the time to retrieve a record because the overflow chain has to be searched as well when the search gets to this leaf. (Although data in the overflow chain can be kept sorted, it usually is not, in order to make inserts fast.) To alleviate this problem, the tree is initially created so that about 20 percent of each page is free. However, once the free space is filled in with inserted records, unless space is freed again through deletes, overflow chains can be eliminated only by a complete reorganization of the file.

The fact that only leaf pages are modified also has an important advantage with respect to concurrent access. When a page is accessed, it is typically 'locked' by the requestor to ensure that it is not concurrently modified by other users of the page. To modify a page, it must be locked in 'exclusive' mode, which is permitted only when no one else holds a lock on the page. Locking can lead to queues of users (*transactions*, to be more precise) waiting to get access to a page. Queues can be a significant performance bottleneck, especially for heavily accessed pages near the root of an index structure. In the ISAM structure, since we know that index-level pages are never modified, we can safely omit the locking step. Not locking index-level pages is an important advantage of ISAM over a dynamic structure like a B+ tree. If the data distribution and size is relatively static, which means overflow chains are rare, ISAM might be preferable to B+ trees due to this advantage.

9.2 B+ TREES: A DYNAMIC INDEX STRUCTURE

A static structure such as the ISAM index suffers from the problem that long overflow chains can develop as the file grows, leading to poor performance. This problem motivated the development of more flexible, dynamic structures that adjust gracefully to inserts and deletes. The **B+ tree** search structure, which is widely used, is a balanced tree in which the internal nodes direct the search and the leaf nodes contain the data entries. Since the tree structure grows and shrinks dynamically, it is not feasible to allocate the leaf pages sequentially as in ISAM, where the set of primary leaf pages was static. In order to retrieve all leaf pages efficiently, we have to link them using page pointers. By organizing them into a doubly linked list, we can easily traverse the sequence of leaf pages (sometimes called the **sequence set**) in either direction. This structure is illustrated in Figure 9.8.

The following are some of the main characteristics of a B+ tree:

- Operations (insert, delete) on the tree keep it balanced.
- A minimum occupancy of 50 percent is guaranteed for each node except the root if the deletion algorithm discussed in Section 9.6 is implemented. However, deletion is often implemented by simply locating the data entry and removing it, without adjusting the tree as needed to guarantee the 50 percent occupancy, because files typically grow rather than shrink.
- Searching for a record requires just a traversal from the root to the appropriate leaf. We will refer to the length of a path from the root to a leaf—any leaf, because

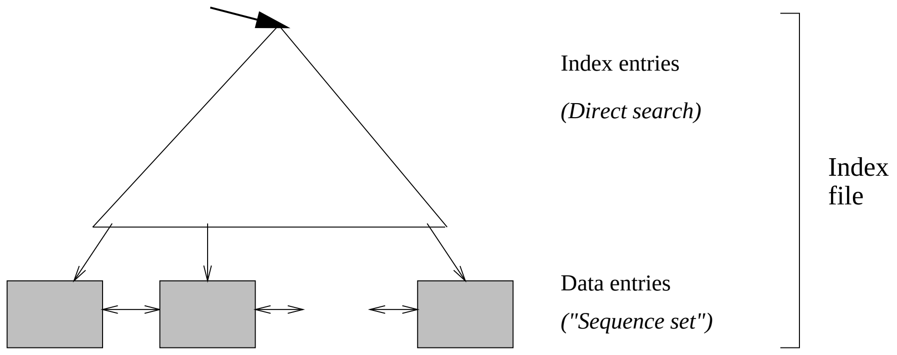


Figure 9.8 Structure of a B+ Tree

the tree is balanced—as the **height** of the tree. For example, a tree with only a leaf level and a single index level, such as the tree shown in Figure 9.10, has height 1. Because of high fan-out, the height of a B+ tree is rarely more than 3 or 4.

We will study B+ trees in which every node contains m entries, where $d \leq m \leq 2d$. The value d is a parameter of the B+ tree, called the **order** of the tree, and is a measure of the capacity of a tree node. The root node is the only exception to this requirement on the number of entries; for the root it is simply required that $1 \leq m \leq 2d$.

If a file of records is updated frequently and sorted access is important, maintaining a B+ tree index with data records stored as data entries is almost always superior to maintaining a sorted file. For the space overhead of storing the index entries, we obtain all the advantages of a sorted file plus efficient insertion and deletion algorithms. B+ trees typically maintain 67 percent space occupancy. B+ trees are usually also preferable to ISAM indexing because inserts are handled gracefully without overflow chains. However, if the dataset size and distribution remain fairly static, overflow chains may not be a major problem. In this case, two factors favor ISAM: the leaf pages are allocated in sequence (making scans over a large range more efficient than in a B+ tree, in which pages are likely to get out of sequence on disk over time, even if they were in sequence after bulk-loading), and the locking overhead of ISAM is lower than that for B+ trees. As a general rule, however, B+ trees are likely to perform better than ISAM.

9.3 FORMAT OF A NODE

The format of a node is the same as for ISAM and is shown in Figure 9.1. Non-leaf nodes with m *index entries* contain $m+1$ pointers to children. Pointer P_i points to a subtree in which all key values K are such that $K_i \leq K < K_{i+1}$. As special cases, P_0 points to a tree in which all key values are less than K_1, and P_m points to a tree in which all key values are greater than or equal to K_m. For leaf nodes, entries are denoted as $k*$, as usual. Just as in ISAM, leaf nodes (and *only* leaf nodes!) contain *data entries.* In the common case that Alternative (2) or (3) is used, leaf entries are $\langle K, I(K) \rangle$ pairs, just like non-leaf entries. Regardless of the alternative chosen for leaf entries, the leaf pages are chained together in a doubly linked list. Thus, the leaves form a sequence, which can be used to answer range queries efficiently.

The reader should carefully consider how such a node organization can be achieved using the record formats presented in Section 7.7; after all, each key–pointer pair can be thought of as a record. If the field being indexed is of fixed length, these index entries will be of fixed length; otherwise, we have variable-length records. In either case the B+ tree can itself be viewed as a file of records. If the leaf pages do not contain the actual data records, then the B+ tree is indeed a file of records that is distinct from the file that contains the data. If the leaf pages contain data records, then a file contains the B+ tree as well as the data.

9.4 SEARCH

The algorithm for search finds the leaf node in which a given data entry belongs. A pseudocode sketch of the algorithm is given in Figure 9.9. We use the notation **ptr* to denote the value pointed to by a pointer variable *ptr* and & *(value)* to denote the address of *value.* Note that finding i in *tree_search* requires us to search within the node, which can be done with either a linear search or a binary search (e.g., depending on the number of entries in the node).

In discussing the search, insertion, and deletion algorithms for B+ trees, we will assume that there are no *duplicates.* That is, no two data entries are allowed to have the same key value. Of course, duplicates arise whenever the search key does not contain a candidate key and must be dealt with in practice. We consider how duplicates can be handled in Section 9.7.

Consider the sample B+ tree shown in Figure 9.10. This B+ tree is of order d=2. That is, each node contains between 2 and 4 entries. Each non-leaf entry is a $\langle$*key value, nodepointer*$\rangle$ pair; at the leaf level, the entries are data records that we denote by $k*$. To search for entry 5*, we follow the left-most child pointer, since $5 < 13$. To search for the entries 14* or 15*, we follow the second pointer, since $13 \leq 14 < 17$, and $13 \leq 15 < 17$. (We don't find 15* on the appropriate leaf, and we can conclude that it is not present in the tree.) To find 24*, we follow the fourth child pointer, since $24 \leq 24 < 30$.

```
func find (search key value $K$) returns nodepointer
// Given a search key value, finds its leaf node
return tree_search(root, $K$);                              // searches from root
endfunc

func tree_search (nodepointer, search key value $K$) returns nodepointer
// Searches tree for entry
if *nodepointer is a leaf, return nodepointer;
else,
    if $K < K_1$ then return tree_search($P_0$, $K$);
    else,
        if $K \geq K_m$ then return tree_search($P_m$, $K$);        // $m$ = # entries
        else,
            find $i$ such that $K_i \leq K < K_{i+1}$;
            return tree_search($P_i$, $K$)
endfunc
```

Figure 9.9 Algorithm for Search

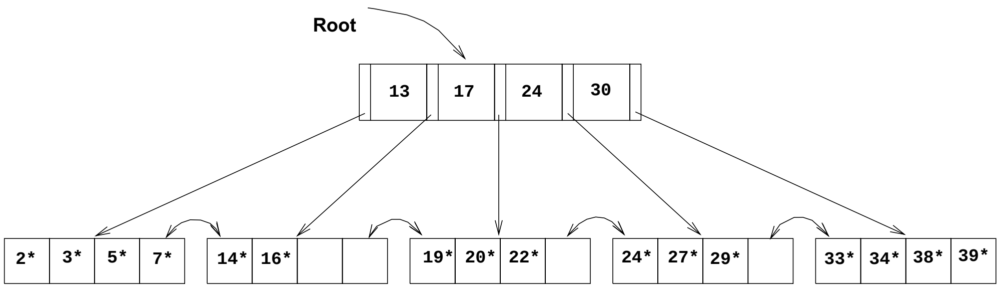


Figure 9.10 Example of a B+ Tree, Order d=2

9.5 INSERT

The algorithm for insertion takes an entry, finds the leaf node where it belongs, and inserts it there. Pseudocode for the B+ tree insertion algorithm is given in Figure 9.11. The basic idea behind the algorithm is that we recursively insert the entry by calling the insert algorithm on the appropriate child node. Usually, this procedure results in going down to the leaf node where the entry belongs, placing the entry there, and returning all the way back to the root node. Occasionally a node is full and it must be split. When the node is split, an entry pointing to the node created by the split must be inserted into its parent; this entry is pointed to by the pointer variable *newchildentry*. If the (old) root is split, a new root node is created and the height of the tree increases by one.

To illustrate insertion, let us continue with the sample tree shown in Figure 9.10. If we insert entry 8*, it belongs in the left-most leaf, which is already full. This insertion causes a split of the leaf page; the split pages are shown in Figure 9.12. The tree must now be adjusted to take the new leaf page into account, so we insert an entry consisting of the pair ⟨*5, pointer to new page*⟩ into the parent node. Notice how the key 5, which discriminates between the split leaf page and its newly created sibling, is 'copied up.' We cannot just 'push up' 5, because every data entry must appear in a leaf page.

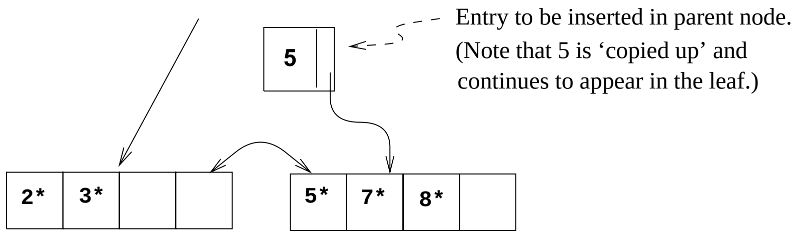
Since the parent node is also full, another split occurs. In general we have to split a non-leaf node when it is full, containing $2d$ keys and $2d + 1$ pointers, and we have to add another index entry to account for a child split. We now have $2d + 1$ keys and $2d + 2$ pointers, yielding two minimally full non-leaf nodes, each containing d keys and $d + 1$ pointers, and an extra key, which we choose to be the 'middle' key. This key and a pointer to the second non-leaf node constitute an index entry that must be inserted into the parent of the split non-leaf node. The middle key is thus 'pushed up' the tree, in contrast to the case for a split of a leaf page.

The split pages in our example are shown in Figure 9.13. The index entry pointing to the new non-leaf node is the pair ⟨*17, pointer to new index-level page*⟩; notice that the key value 17 is 'pushed up' the tree, in contrast to the splitting key value 5 in the leaf split, which was 'copied up.'

The difference in handling leaf-level and index-level splits arises from the B+ tree requirement that all data entries $k*$ must reside in the leaves. This requirement prevents us from 'pushing up' 5 and leads to the slight redundancy of having some key values appearing in the leaf level as well as in some index level. However, range queries can be efficiently answered by just retrieving the sequence of leaf pages; the redundancy is a small price to pay for efficiency. In dealing with the index levels, we have more flexibility, and we 'push up' 17 to avoid having two copies of 17 in the index levels.

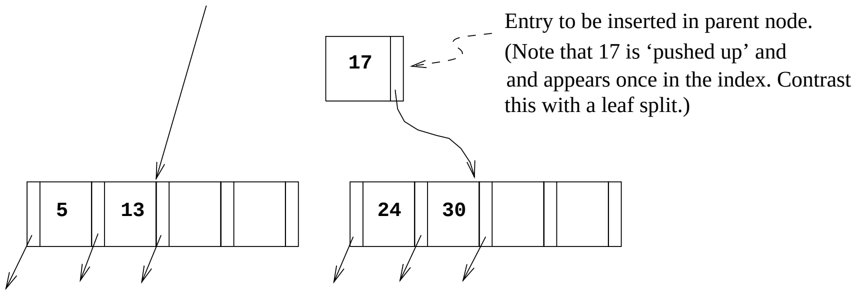
```
proc insert (nodepointer, entry, newchildentry)
// Inserts entry into subtree with root '*nodepointer'; degree is d;
// 'newchildentry' is null initially, and null upon return unless child is split

if *nodepointer is a non-leaf node, say $N$,
    find $i$ such that $K_i \leq$ entry's key value $< K_{i+1}$;          // choose subtree
    insert($P_i$, entry, newchildentry);                        // recursively, insert entry
    if newchildentry is null, return;                  // usual case; didn't split child
    else,                          // we split child, must insert *newchildentry in $N$
        if $N$ has space,                                                  // usual case
            put *newchildentry on it, set newchildentry to null, return;
        else,                              // note difference wrt splitting of leaf page!
            split $N$:                      // $2d+1$ key values and $2d+2$ nodepointers
            first $d$ key values and $d+1$ nodepointers stay,
            last $d$ keys and $d+1$ pointers move to new node, $N2$;
            // *newchildentry set to guide searches between $N$ and $N2$
            newchildentry = & (⟨smallest key value on $N2$, pointer to N2⟩);
            if $N$ is the root,                                // root node was just split
                create new node with ⟨pointer to $N$, *newchildentry⟩;
                make the tree's root-node pointer point to the new node;
            return;

if *nodepointer is a leaf node, say $L$,
    if $L$ has space,                                                      // usual case
    put entry on it, set newchildentry to null, and return;
    else,                                          // once in a while, the leaf is full
        split $L$: first $d$ entries stay, rest move to brand new node $L2$;
        newchildentry = & (⟨smallest key value on $L2$, pointer to $L2$⟩);
        set sibling pointers in $L$ and $L2$;
        return;
endproc
```

Figure 9.11 Algorithm for Insertion into B+ Tree of Order d



5
Entry to be inserted in parent node.
(Note that 5 is 'copied up' and
continues to appear in the leaf.)
2*
3*
5*
7*
8*

Figure 9.12 Split Leaf Pages during Insert of Entry 8*



17
Entry to be inserted in parent node.
(Note that 17 is 'pushed up' and
and appears once in the index. Contrast
this with a leaf split.)
5
13
24
30

Figure 9.13 Split Index Pages during Insert of Entry 8*

Now, since the split node was the old root, we need to create a new root node to hold the entry that distinguishes the two split index pages. The tree after completing the insertion of the entry 8* is shown in Figure 9.14.

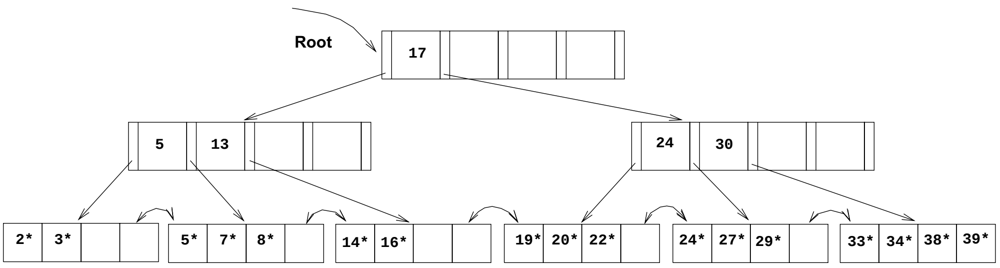


Figure 9.14 B+ Tree after Inserting Entry 8*

One variation of the insert algorithm tries to redistribute entries of a node N with a sibling before splitting the node; this improves average occupancy. The **sibling** of a node N, in this context, is a node that is immediately to the left or right of N *and has the same parent as N.*

To illustrate redistribution, reconsider insertion of entry 8* into the tree shown in Figure 9.10. The entry belongs in the left-most leaf, which is full. However, the (only) sibling of this leaf node contains only two entries and can thus accommodate more entries. We can therefore handle the insertion of 8* with a redistribution. Note how the entry in the parent node that points to the second leaf has a new key value; we 'copy up' the new low key value on the second leaf. This process is illustrated in Figure 9.15.

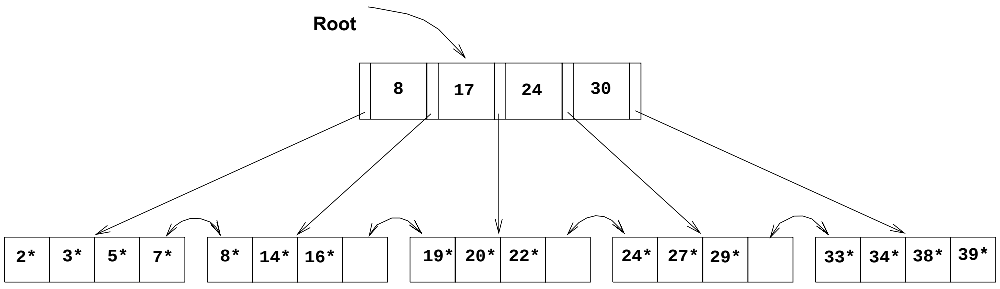


Figure 9.15 B+ Tree after Inserting Entry 8* Using Redistribution

To determine whether redistribution is possible, we have to retrieve the sibling. If the sibling happens to be full, we have to split the node anyway. On average, checking whether redistribution is possible increases I/O for index node splits, especially if we check both siblings. (Checking whether redistribution is possible may reduce I/O if the redistribution succeeds whereas a split propagates up the tree, but this case is very infrequent.) If the file is growing, average occupancy will probably not be affected much even if we do not redistribute. Taking these considerations into account, *not* redistributing entries at non-leaf levels usually pays off.

If a split occurs at the leaf level, however, we have to retrieve a neighbor in order to adjust the previous and next-neighbor pointers with respect to the newly created leaf node. Therefore, a limited form of redistribution makes sense: If a leaf node is full, fetch a neighbor node; if it has space, and has the same parent, redistribute entries. Otherwise (neighbor has different parent, i.e., is not a sibling, or is also full) split the leaf node and adjust the previous and next-neighbor pointers in the split node, the newly created neighbor, and the old neighbor.

9.6 DELETE *

The algorithm for deletion takes an entry, finds the leaf node where it belongs, and deletes it. Pseudocode for the B+ tree deletion algorithm is given in Figure 9.16. The basic idea behind the algorithm is that we recursively delete the entry by calling the delete algorithm on the appropriate child node. We usually go down to the leaf node where the entry belongs, remove the entry from there, and return all the way back to the root node. Occasionally a node is at minimum occupancy before the deletion, and the deletion causes it to go below the occupancy threshold. When this happens, we must either redistribute entries from an adjacent sibling or merge the node with a sibling to maintain minimum occupancy. If entries are redistributed between two nodes, their parent node must be updated to reflect this; the key value in the index entry pointing to the second node must be changed to be the lowest search key in the second node. If two nodes are merged, their parent must be updated to reflect this by deleting the index entry for the second node; this index entry is pointed to by the pointer variable *oldchildentry* when the delete call returns to the parent node. If the last entry in the root node is deleted in this manner because one of its children was deleted, the height of the tree decreases by one.

To illustrate deletion, let us consider the sample tree shown in Figure 9.14. To delete entry 19*, we simply remove it from the leaf page on which it appears, and we are done because the leaf still contains two entries. If we subsequently delete 20*, however, the leaf contains only one entry after the deletion. The (only) sibling of the leaf node that contained 20* has three entries, and we can therefore deal with the situation by redistribution; we move entry 24* to the leaf page that contained 20* and 'copy up' the new splitting key (27, which is the new low key value of the leaf from which we borrowed 24*) into the parent. This process is illustrated in Figure 9.17.

Suppose that we now delete entry 24*. The affected leaf contains only one entry (22*) after the deletion, and the (only) sibling contains just two entries (27* and 29*). Therefore, we cannot redistribute entries. However, these two leaf nodes together contain only three entries and can be merged. While merging, we can 'toss' the entry (⟨*27, pointer to second leaf page*⟩) in the parent, which pointed to the second leaf page, because the second leaf page is empty after the merge and can be discarded. The right subtree of Figure 9.17 after this step in the deletion of entry 24* is shown in Figure 9.18.

Deleting the entry ⟨*27, pointer to second leaf page*⟩ has created a non-leaf-level page with just one entry, which is below the minimum of d=2. To fix this problem, we must either redistribute or merge. In either case we must fetch a sibling. The only sibling of this node contains just two entries (with key values 5 and 13), and so redistribution is not possible; we must therefore merge.

The situation when we have to merge two non-leaf nodes is exactly the opposite of the situation when we have to split a non-leaf node. We have to split a non-leaf node when it contains $2d$ keys and $2d+1$ pointers, and we have to add another key–pointer pair. Since we resort to merging two non-leaf nodes only when we cannot redistribute entries between them, the two nodes must be minimally full; that is, each must contain d keys and $d+1$ pointers prior to the deletion. After merging the two nodes and removing the key–pointer pair to be deleted, we have $2d-1$ keys and $2d+1$ pointers: Intuitively, the left-most pointer on the second merged node lacks a key value. To see what key value must be combined with this pointer to create a complete index entry, consider the parent of the two nodes being merged. The index entry pointing to one of the merged

```
proc delete (parentpointer, nodepointer, entry, oldchildentry)
// Deletes entry from subtree with root '*nodepointer'; degree is d;
// 'oldchildentry' null initially, and null upon return unless child deleted
if *nodepointer is a non-leaf node, say $N$,
    find $i$ such that $K_i \leq$ entry's key value $< K_{i+1}$;           // choose subtree
    delete(nodepointer, $P_i$, entry, oldchildentry);                 // recursive delete
    if oldchildentry is null, return;                // usual case: child not deleted
    else,                                  // we discarded child node (see discussion)
        remove *oldchildentry from $N$,          // next, check minimum occupancy
        if $N$ has entries to spare,                                   // usual case
            set oldchildentry to null, return;           // delete doesn't go further
        else,                               // note difference wrt merging of leaf pages!
            get a sibling $S$ of $N$:               // parentpointer arg used to find $S$
            if $S$ has extra entries,
                redistribute evenly between $N$ and $S$ through parent;
                set oldchildentry to null, return;
            else, merge $N$ and $S$                            // call node on rhs $M$
                oldchildentry = & (current entry in parent for $M$);
                pull splitting key from parent down into node on left;
                move all entries from $M$ to node on left;
                discard empty node $M$, return;

if *nodepointer is a leaf node, say $L$,
    if $L$ has entries to spare,                                        // usual case
        remove entry, set oldchildentry to null, and return;
    else,                                // once in a while, the leaf becomes underfull
        get a sibling $S$ of $L$;                       // parentpointer used to find $S$
        if $S$ has extra entries,
            redistribute evenly between $L$ and $S$;
            find entry in parent for node on right;                     // call it $M$
            replace key value in parent entry by new low-key value in $M$;
            set oldchildentry to null, return;
        else, merge $L$ and $S$                                // call node on rhs $M$
            oldchildentry = & (current entry in parent for $M$);
            move all entries from $M$ to node on left;
            discard empty node $M$, adjust sibling pointers, return;
endproc
```

Figure 9.16 Algorithm for Deletion from B+ Tree of Order d

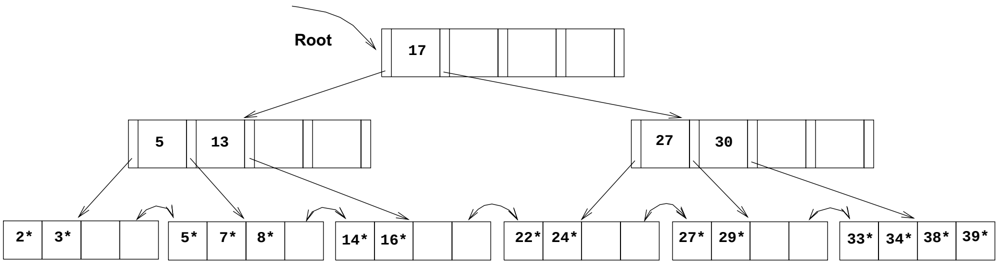


Figure 9.17 B+ Tree after Deleting Entries 19* and 20*

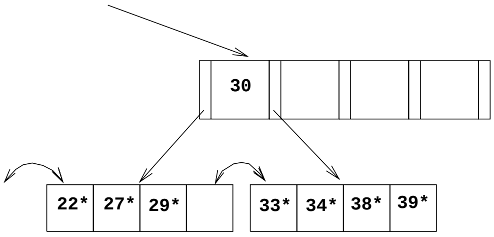


Figure 9.18 Partial B+ Tree during Deletion of Entry 24*

nodes must be deleted from the parent because the node is about to be discarded. The key value in this index entry is precisely the key value we need to complete the new merged node: The entries in the first node being merged, followed by the splitting key value that is 'pulled down' from the parent, followed by the entries in the second non-leaf node gives us a total of $2d$ keys and $2d + 1$ pointers, which is a full non-leaf node. Notice how the splitting key value in the parent is 'pulled down,' in contrast to the case of merging two leaf nodes.

Consider the merging of two non-leaf nodes in our example. Together, the non-leaf node and the sibling to be merged contain only three entries, and they have a total of five pointers to leaf nodes. To merge the two nodes, we also need to 'pull down' the index entry in their parent that currently discriminates between these nodes. This index entry has key value 17, and so we create a new entry $\langle$*17, left-most child pointer in sibling*$\rangle$. Now we have a total of four entries and five child pointers, which can fit on one page in a tree of order d=2. Notice that pulling down the splitting key 17 means that it will no longer appear in the parent node following the merge. After we merge the affected non-leaf node and its sibling by putting all the entries on one page and discarding the empty sibling page, the new node is the only child of the old root, which can therefore be discarded. The tree after completing all these steps in the deletion of entry 24* is shown in Figure 9.19.

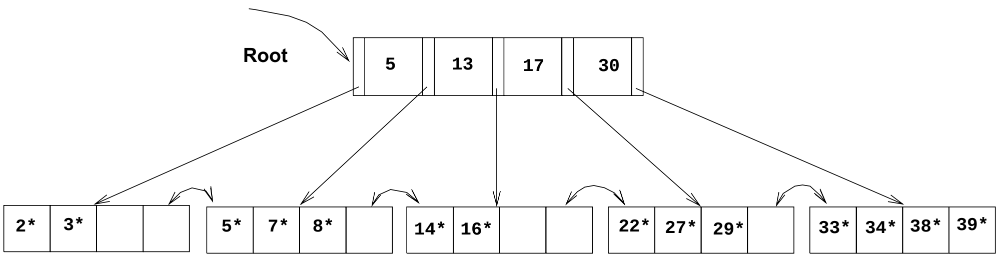


Figure 9.19 B+ Tree after Deleting Entry 24*

The previous examples illustrated redistribution of entries across leaves and merging of both leaf-level and non-leaf-level pages. The remaining case is that of redistribution of entries between non-leaf-level pages. To understand this case, consider the intermediate right subtree shown in Figure 9.18. We would arrive at the same intermediate right subtree if we try to delete 24* from a tree similar to the one shown in Figure 9.17 but with the left subtree and root key value as shown in Figure 9.20. The tree in Figure 9.20 illustrates an intermediate stage during the deletion of 24*. (Try to construct the initial tree.)

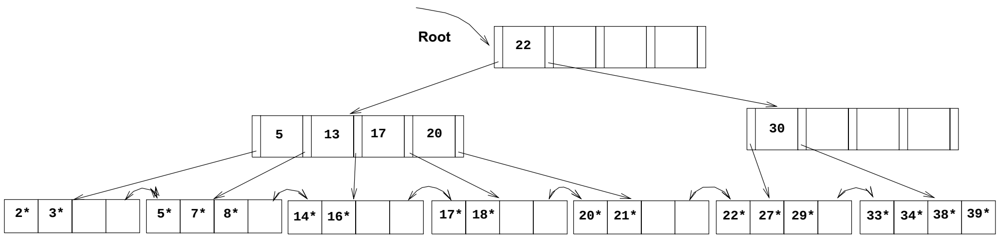


Figure 9.20 A B+ Tree during a Deletion

In contrast to the case when we deleted 24* from the tree of Figure 9.17, the non-leaf level node containing key value 30 now has a sibling that can spare entries (the entries with key values 17 and 20). We move these entries[2] over from the sibling. Notice that in doing so, we essentially 'push' them through the splitting entry in their parent node (the root), which takes care of the fact that 17 becomes the new low key value on the right and therefore must replace the old splitting key in the root (the key value 22). The tree with all these changes is shown in Figure 9.21.

In concluding our discussion of deletion, we note that we retrieve only one sibling of a node. If this node has spare entries, we use redistribution; otherwise, we merge. If the node has a second sibling, it may be worth retrieving that sibling as well to

[2]It is sufficient to move over just the entry with key value 20, but we are moving over two entries to illustrate what happens when several entries are redistributed.

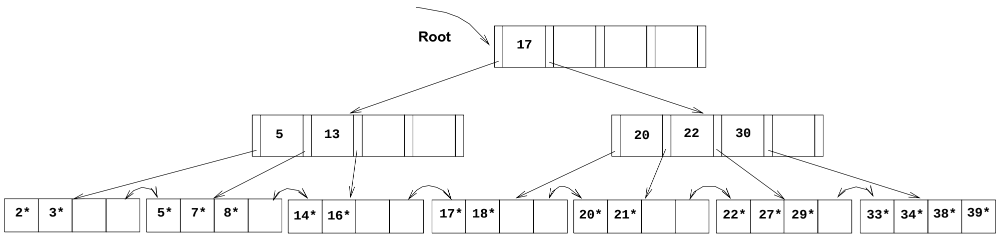


Figure 9.21 B+ Tree after Deletion

check for the possibility of redistribution. Chances are high that redistribution will be possible, and unlike merging, redistribution is guaranteed to propagate no further than the parent node. Also, the pages have more space on them, which reduces the likelihood of a split on subsequent insertions. (Remember, files typically grow, not shrink!) However, the number of times that this case arises (node becomes less than half-full and first sibling can't spare an entry) is not very high, so it is not essential to implement this refinement of the basic algorithm that we have presented.

9.7 DUPLICATES *

The search, insertion, and deletion algorithms that we have presented ignore the issue of **duplicate keys**, that is, several data entries with the same key value. We now discuss how duplicates can be handled.

The basic search algorithm assumes that all entries with a given key value reside on a single leaf page. One way to satisfy this assumption is to use *overflow pages* to deal with duplicates. (In ISAM, of course, we have overflow pages in any case, and duplicates are easily handled.)

Typically, however, we use an alternative approach for duplicates. We handle them just like any other entries and several leaf pages may contain entries with a given key value. To retrieve all data entries with a given key value, we must search for the *left-most* data entry with the given key value and then possibly retrieve more than one leaf page (using the leaf sequence pointers). Modifying the search algorithm to find the left-most data entry in an index with duplicates is an interesting exercise (in fact, it is Exercise 9.11).

One problem with this approach is that when a record is deleted, if we use Alternative (2) for data entries, finding the corresponding data entry to delete in the B+ tree index could be inefficient because we may have to check several duplicate entries ⟨*key, rid*⟩ with the same *key* value. This problem can be addressed by considering the *rid* value in the data entry to be *part of the search key*, for purposes of positioning the data

Duplicate handling in commercial systems: In a clustered index in Sybase ASE, the data rows are maintained in sorted order on the page and in the collection of data pages. The data pages are bidirectionally linked in sort order. Rows with duplicate keys are inserted into (or deleted from) the ordered set of rows. This may result in overflow pages of rows with duplicate keys being inserted into the page chain or empty overflow pages removed from the page chain. Insertion or deletion of a duplicate key does not affect the higher index levels unless a split or merge of a non-overflow page occurs. In IBM DB2, Oracle 8, and Microsoft SQL Server, duplicates are handled by adding a row id if necessary to eliminate duplicate key values.

entry in the tree. This solution effectively turns the index into a *unique* index (i.e., no duplicates). Remember that a search key can be any sequence of fields—in this variant, the rid of the data record is essentially treated as another field while constructing the search key.

Alternative (3) for data entries leads to a natural solution for duplicates, but if we have a large number of duplicates, a single data entry could span multiple pages. And of course, when a data record is deleted, finding the rid to delete from the corresponding data entry can be inefficient. The solution to this problem is similar to the one discussed above for Alternative (2): We can maintain the list of rids within each data entry in sorted order (say, by page number and then slot number if a rid consists of a page id and a slot id).

9.8 B+ TREES IN PRACTICE *

In this section we discuss several important pragmatic issues.

9.8.1 Key Compression

The height of a B+ tree depends on the *number of data entries* and the *size of index entries*. The size of index entries determines the number of index entries that will fit on a page and, therefore, the *fan-out* of the tree. Since the height of the tree is proportional to $log_{fan-out}$*(# of data entries)*, and the number of disk I/Os to retrieve a data entry is equal to the height (unless some pages are found in the buffer pool) it is clearly important to maximize the fan-out, to minimize the height.

An index entry contains a search key value and a page pointer. Thus the size primarily depends on the size of the search key value. If search key values are very long (for instance, the name Devarakonda Venkataramana Sathyanarayana Seshasayee Yella-

B+ Trees in Real Systems: IBM DB2, Informix, Microsoft SQL Server, Oracle 8, and Sybase ASE all support clustered and unclustered B+ tree indexes, with some differences in how they handle deletions and duplicate key values. In Sybase ASE, depending on the concurrency control scheme being used for the index, the deleted row is removed (with merging if the page occupancy goes below threshold) or simply marked as deleted; a garbage collection scheme is used to recover space in the latter case. In Oracle 8, deletions are handled by marking the row as deleted. To reclaim the space occupied by deleted records, we can rebuild the index online (i.e., while users continue to use the index) or coalesce underfull pages (which does not reduce tree height). Coalesce is in-place, rebuild creates a copy. Informix handles deletions by marking simply marking records as deleted. DB2 and SQL Server remove deleted records and merge pages when occupancy goes below threshold.
Oracle 8 also allows records from multiple relations to be co-clustered on the same page. The co-clustering can be based on a B+ tree search key or static hashing and upto 32 relns can be stored together.

manchali Murthy), not many index entries will fit on a page; fan-out is low, and the height of the tree is large.

On the other hand, search key values in index entries are used only to direct traffic to the appropriate leaf. When we want to locate data entries with a given search key value, we compare this search key value with the search key values of index entries (on a path from the root to the desired leaf). During the comparison at an index-level node, we want to identify two index entries with search key values k_1 and k_2 such that the desired search key value k falls between k_1 and k_2. To accomplish this, we do not need to store search key values in their entirety in index entries.

For example, suppose that we have two adjacent index entries in a node, with search key values 'David Smith' and 'Devarakonda ...' To discriminate between these two values, it is sufficient to store the abbreviated forms 'Da' and 'De.' More generally, the meaning of the entry 'David Smith' in the B+ tree is that every value in the subtree pointed to by the pointer to the left of 'David Smith' is less than 'David Smith,' and every value in the subtree pointed to by the pointer to the right of 'David Smith' is (greater than or equal to 'David Smith' and) less than 'Devarakonda ...'

To ensure that this semantics for an entry is preserved, while compressing the entry with key 'David Smith,' we must examine the largest key value in the subtree to the left of 'David Smith' and the smallest key value in the subtree to the right of 'David Smith,' not just the index entries ('Daniel Lee' and 'Devarakonda ...') that are its neighbors. This point is illustrated in Figure 9.22; the value 'Davey Jones' is greater than 'Dav,' and thus, 'David Smith' can only be abbreviated to 'Davi,' not to 'Dav.'

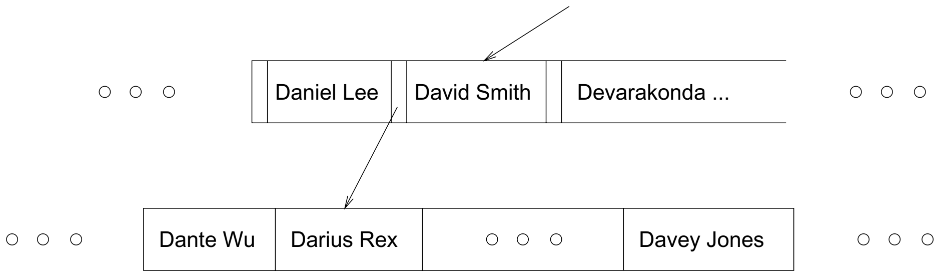


Figure 9.22 Example Illustrating Prefix Key Compression

This technique is called **prefix key compression**, or simply **key compression**, and is supported in many commercial implementations of B+ trees. It can substantially increase the fan-out of a tree. We will not discuss the details of the insertion and deletion algorithms in the presence of key compression.

9.8.2 Bulk-Loading a B+ Tree

Entries are added to a B+ tree in two ways. First, we may have an existing collection of data records with a B+ tree index on it; whenever a record is added to the collection, a corresponding entry must be added to the B+ tree as well. (Of course, a similar comment applies to deletions.) Second, we may have a collection of data records for which we want to create a B+ tree index on some key field(s). In this situation, we can start with an empty tree and insert an entry for each data record, one at a time, using the standard insertion algorithm. However, this approach is likely to be quite expensive because each entry requires us to start from the root and go down to the appropriate leaf page. Even though the index-level pages are likely to stay in the buffer pool between successive requests, the overhead is still considerable.

For this reason many systems provide a *bulk-loading* utility for creating a B+ tree index on an existing collection of data records. The first step is to sort the data entries $k*$ to be inserted into the (to be created) B+ tree according to the search key k. (If the entries are key–pointer pairs, sorting them does not mean sorting the data records that are pointed to, of course.) We will use a running example to illustrate the bulk-loading algorithm. We will assume that each data page can hold only two entries, and that each index page can hold two entries and an additional pointer (i.e., the B+ tree is assumed to be of order d=1).

After the data entries have been sorted, we allocate an empty page to serve as the root and insert a pointer to the first page of (sorted) entries into it. We illustrate this process in Figure 9.23, using a sample set of nine sorted pages of data entries.

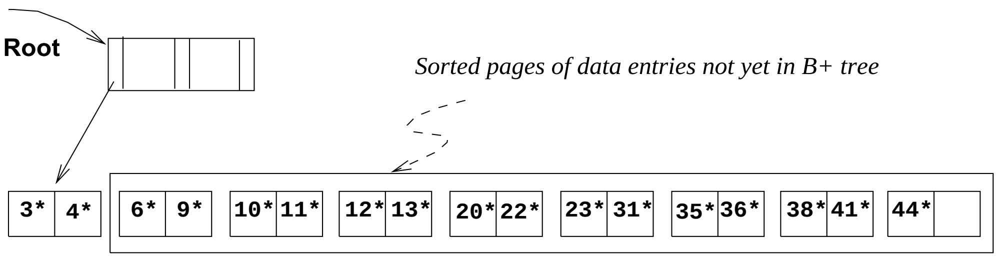


Figure 9.23 Initial Step in B+ Tree Bulk-Loading

We then add one entry to the root page for each page of the sorted data entries. The new entry consists of ⟨*low key value on page, pointer to page*⟩. We proceed until the root page is full; see Figure 9.24.

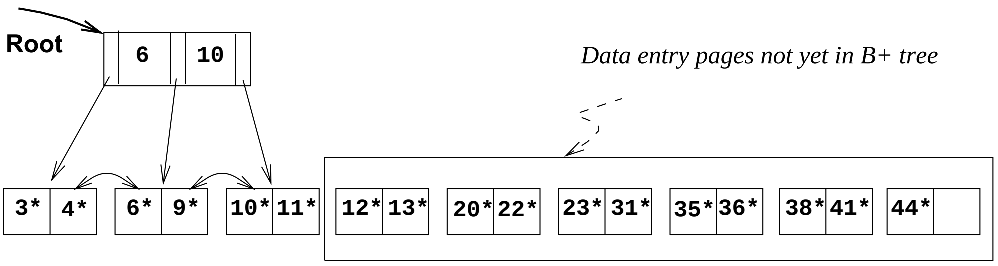


Figure 9.24 Root Page Fills up in B+ Tree Bulk-Loading

To insert the entry for the next page of data entries, we must split the root and create a new root page. We show this step in Figure 9.25.

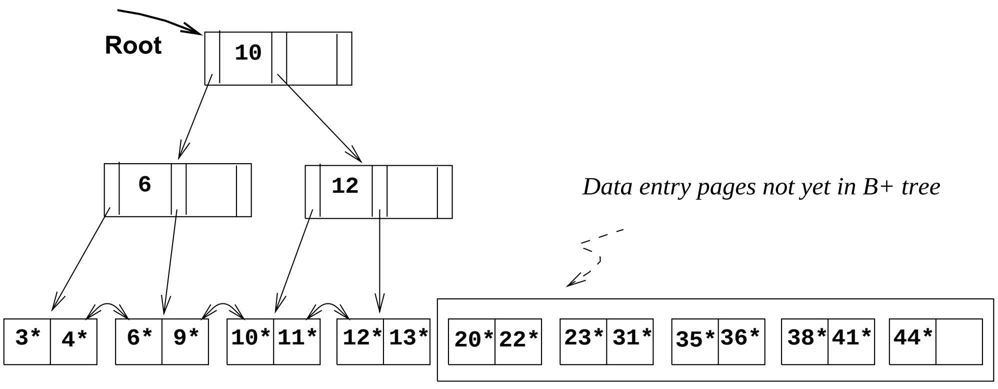


Figure 9.25 Page Split during B+ Tree Bulk-Loading

We have redistributed the entries evenly between the two children of the root, in anticipation of the fact that the B+ tree is likely to grow. Although it is difficult (!) to illustrate these options when at most two entries fit on a page, we could also have just left all the entries on the old page or filled up some desired fraction of that page (say, 80 percent). These alternatives are simple variants of the basic idea.

To continue with the bulk-loading example, entries for the leaf pages are always inserted into the right-most index page just above the leaf level. When the right-most index page above the leaf level fills up, it is split. This action may cause a split of the right-most index page one step closer to the root, as illustrated in Figures 9.26 and 9.27.

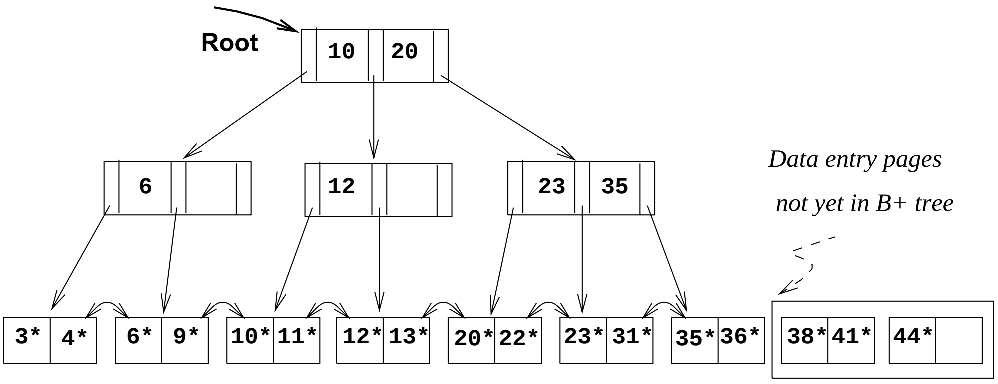


Figure 9.26 Before Adding Entry for Leaf Page Containing 38*

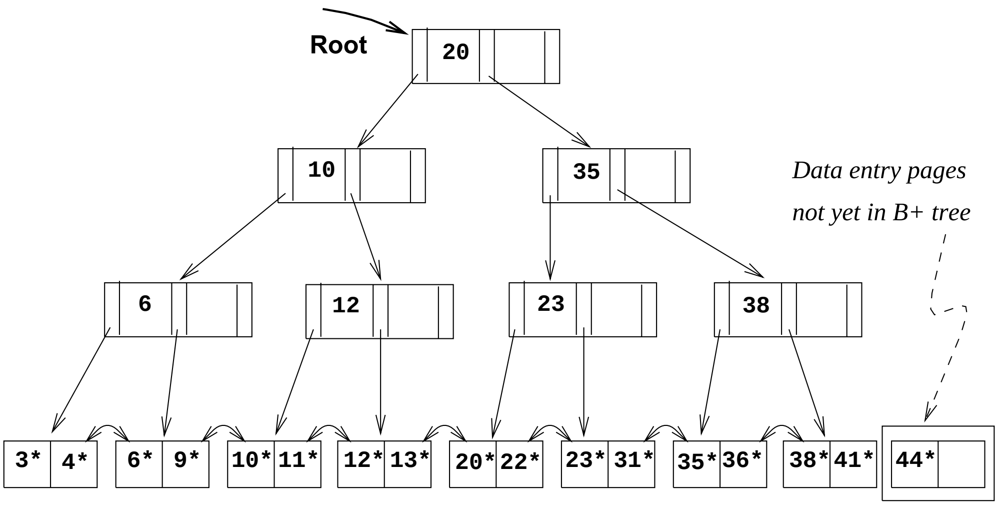


Figure 9.27 After Adding Entry for Leaf Page Containing 38*

Note that splits occur only on the right-most path from the root to the leaf level. We leave the completion of the bulk-loading example as a simple exercise.

Let us consider the cost of creating an index on an existing collection of records. This operation consists of three steps: (1) creating the data entries to insert in the index, (2) sorting the data entries, and (3) building the index from the sorted entries. The first step involves scanning the records and writing out the corresponding data entries; the cost is $(R + E)$ I/Os, where R is the number of pages containing records and E is the number of pages containing data entries. Sorting is discussed in Chapter 11; you will see that the index entries can be generated in sorted order at a cost of about $3E$ I/Os. These entries can then be inserted into the index as they are generated, using the bulk-loading algorithm discussed in this section. The cost of the third step, that is, inserting the entries into the index, is then just the cost of writing out all index pages.

9.8.3 The Order Concept

We have presented B+ trees using the parameter d to denote minimum occupancy. It is worth noting that the concept of *order* (i.e., the parameter d), while useful for teaching B+ tree concepts, must usually be relaxed in practice and replaced by a physical space criterion; for example, that nodes must be kept at least half-full.

One reason for this is that leaf nodes and non-leaf nodes can usually hold different numbers of entries. Recall that B+ tree nodes are disk pages and that non-leaf nodes contain only search keys and node pointers, while leaf nodes can contain the actual data records. Obviously, the size of a data record is likely to be quite a bit larger than the size of a search entry, so many more search entries than records will fit on a disk page.

A second reason for relaxing the order concept is that the search key may contain a character string field (e.g., the *name* field of Students) whose size varies from record to record; such a search key leads to variable-size data entries and index entries, and the number of entries that will fit on a disk page becomes variable.

Finally, even if the index is built on a fixed-size field, several records may still have the same search key value (e.g., several Students records may have the same *gpa* or *name* value). This situation can also lead to variable-size leaf entries (if we use Alternative (3) for data entries). Because of all of these complications, the concept of order is typically replaced by a simple physical criterion (e.g., merge if possible when more than half of the space in the node is unused).

9.8.4 The Effect of Inserts and Deletes on Rids

If the leaf pages contain data records—that is, the B+ tree is a clustered index—then operations such as splits, merges, and redistributions can change rids. Recall that a typical representation for a rid is some combination of (physical) page number and slot number. This scheme allows us to move records within a page if an appropriate page format is chosen, but not across pages, as is the case with operations such as splits. So unless rids are chosen to be independent of page numbers, an operation such as split or merge in a clustered B+ tree may require compensating updates to other indexes on the same data.

A similar comment holds for any dynamic clustered index, regardless of whether it is tree-based or hash-based. Of course, the problem does not arise with nonclustered indexes because only index entries are moved around.

9.9 POINTS TO REVIEW

- Tree-structured indexes are ideal for range selections, and also support equality selections quite efficiently. *ISAM* is a static tree-structured index in which only leaf pages are modified by inserts and deletes. If a leaf page is full, an *overflow page* is added. Unless the size of the dataset and the data distribution remain approximately the same, overflow chains could become long and degrade performance. **(Section 9.1)**
- A B+ tree is a dynamic, height-balanced index structure that adapts gracefully to changing data characteristics. Each node except the root has between d and $2d$ entries. The number d is called the *order* of the tree. **(Section 9.2)**
- Each non-leaf node with m index entries has $m+1$ children pointers. The leaf nodes contain data entries. Leaf pages are chained in a doubly linked list. **(Section 9.3)**
- An equality search requires traversal from the root to the corresponding leaf node of the tree. **(Section 9.4)**
- During insertion, nodes that are full are *split* to avoid overflow pages. Thus, an insertion might increase the height of the tree. **(Section 9.5)**
- During deletion, a node might go below the minimum occupancy threshold. In this case, we can either redistribute entries from adjacent siblings, or we can merge the node with a sibling node. A deletion might decrease the height of the tree. **(Section 9.6)**
- Duplicate search keys require slight modifications to the basic B+ tree operations. **(Section 9.7)**

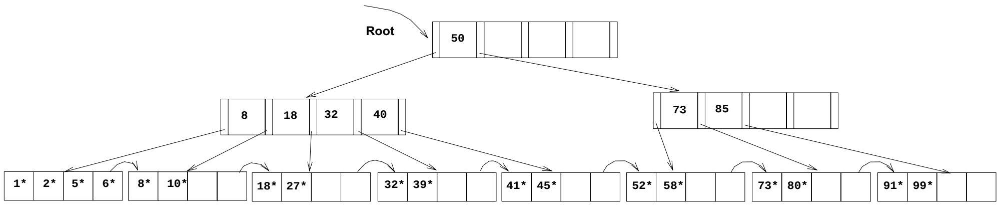


Figure 9.28 Tree for Exercise 9.1

- In *key compression*, search key values in index nodes are shortened to ensure a high fan-out. A new B+ tree index can be efficiently constructed for a set of records using a *bulk-loading* procedure. In practice, the concept of order is replaced by a physical space criterion. **(Section 9.8)**

EXERCISES

Exercise 9.1 Consider the B+ tree index of order $d = 2$ shown in Figure 9.28.

1. Show the tree that would result from inserting a data entry with key 9 into this tree.
2. Show the B+ tree that would result from inserting a data entry with key 3 into the original tree. How many page reads and page writes will the insertion require?
3. Show the B+ tree that would result from deleting the data entry with key 8 from the original tree, assuming that the left sibling is checked for possible redistribution.
4. Show the B+ tree that would result from deleting the data entry with key 8 from the original tree, assuming that the right sibling is checked for possible redistribution.
5. Show the B+ tree that would result from starting with the original tree, inserting a data entry with key 46 and then deleting the data entry with key 52.
6. Show the B+ tree that would result from deleting the data entry with key 91 from the original tree.
7. Show the B+ tree that would result from starting with the original tree, inserting a data entry with key 59, and then deleting the data entry with key 91.
8. Show the B+ tree that would result from successively deleting the data entries with keys 32, 39, 41, 45, and 73 from the original tree.

Exercise 9.2 Consider the B+ tree index shown in Figure 9.29, which uses Alternative (1) for data entries. Each intermediate node can hold up to five pointers and four key values. Each leaf can hold up to four records, and leaf nodes are doubly linked as usual, although these links are not shown in the figure.

Answer the following questions.

1. Name all the tree nodes that must be fetched to answer the following query: "Get all records with search key greater than 38."

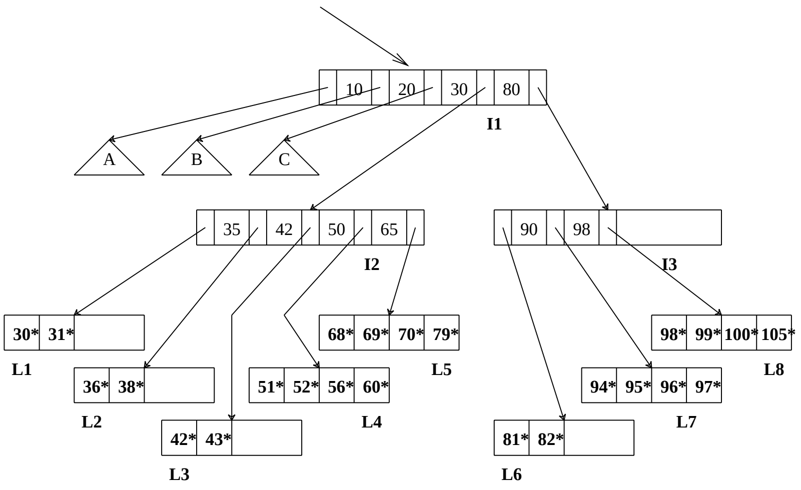


Figure 9.29 Tree for Exercise 9.2

2. Insert a record with search key 109 into the tree.
3. Delete the record with search key 81 from the (original) tree.
4. Name a search key value such that inserting it into the (original) tree would cause an increase in the height of the tree.
5. Note that subtrees A, B, and C are not fully specified. Nonetheless, what can you infer about the contents and the shape of these trees?
6. How would your answers to the above questions change if this were an ISAM index?
7. Suppose that this is an ISAM index. What is the minimum number of insertions needed to create a chain of three overflow pages?

Exercise 9.3 Answer the following questions.

1. What is the minimum space utilization for a B+ tree index?
2. What is the minimum space utilization for an ISAM index?
3. If your database system supported both a static and a dynamic tree index (say, ISAM and B+ trees), would you ever consider using the *static* index in preference to the *dynamic* index?

Exercise 9.4 Suppose that a page can contain at most four data values and that all data values are integers. Using only B+ trees of order 2, give examples of each of the following:

1. A B+ tree whose height changes from 2 to 3 when the value 25 is inserted. Show your structure before and after the insertion.
2. A B+ tree in which the deletion of the value 25 leads to a redistribution. Show your structure before and after the deletion.

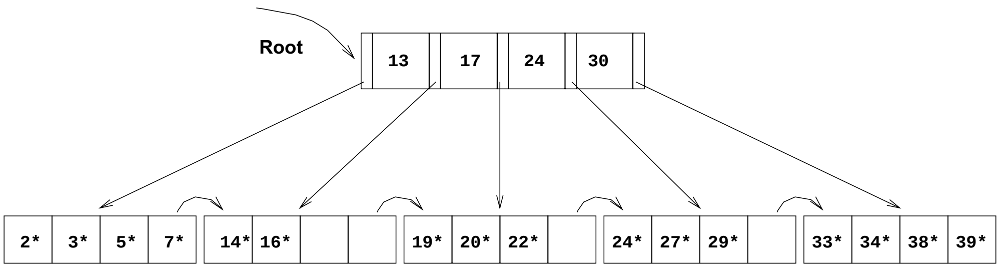


Figure 9.30 Tree for Exercise 9.5

3. A B+ tree in which the deletion of the value 25 causes a merge of two nodes, but without altering the height of the tree.
4. An ISAM structure with four buckets, none of which has an overflow page. Further, every bucket has space for exactly one more entry. Show your structure before and after inserting two additional values, chosen so that an overflow page is created.

Exercise 9.5 Consider the B+ tree shown in Figure 9.30.

1. Identify a list of five data entries such that:
   (a) Inserting the entries in the order shown and then deleting them in the opposite order (e.g., insert *a*, insert *b*, delete *b*, delete *a*) results in the original tree.
   (b) Inserting the entries in the order shown and then deleting them in the opposite order (e.g., insert *a*, insert *b*, delete *b*, delete *a*) results in a different tree.
2. What is the minimum number of insertions of data entries with distinct keys that will cause the height of the (original) tree to change from its current value (of 1) to 3?
3. Would the minimum number of insertions that will cause the original tree to increase to height 3 change if you were allowed to insert duplicates (multiple data entries with the same key), assuming that overflow pages are not used for handling duplicates?

Exercise 9.6 Answer Exercise 9.5 assuming that the tree is an ISAM tree! (Some of the examples asked for may not exist—if so, explain briefly.)

Exercise 9.7 Suppose that you have a sorted file, and you want to construct a dense primary B+ tree index on this file.

1. One way to accomplish this task is to scan the file, record by record, inserting each one using the B+ tree insertion procedure. What performance and storage utilization problems are there with this approach?
2. Explain how the bulk-loading algorithm described in the text improves upon the above scheme.

Exercise 9.8 Assume that you have just built a dense B+ tree index using Alternative (2) on a heap file containing 20,000 records. The key field for this B+ tree index is a 40-byte string, and it is a candidate key. Pointers (i.e., record ids and page ids) are (at most) 10-byte values. The size of one disk page is 1,000 bytes. The index was built in a bottom-up fashion using the bulk-loading algorithm, and the nodes at each level were filled up as much as possible.

1. How many levels does the resulting tree have?
2. For each level of the tree, how many nodes are at that level?
3. How many levels would the resulting tree have if key compression is used and it reduces the average size of each key in an entry to 10 bytes?
4. How many levels would the resulting tree have without key compression, but with all pages 70 percent full?

Exercise 9.9 The algorithms for insertion and deletion into a B+ tree are presented as recursive algorithms. In the code for *insert*, for instance, there is a call made at the parent of a node N to insert into (the subtree rooted at) node N, and when this call returns, the current node is the parent of N. Thus, we do not maintain any 'parent pointers' in nodes of B+ tree. Such pointers are not part of the B+ tree structure for a good reason, as this exercise will demonstrate. An alternative approach that uses parent pointers—again, remember that such pointers are *not* part of the standard B+ tree structure!—in each node appears to be simpler:

> Search to the appropriate leaf using the search algorithm; then insert the entry and split if necessary, with splits propagated to parents if necessary (using the parent pointers to find the parents).

Consider this (unsatisfactory) alternative approach:

1. Suppose that an internal node N is split into nodes N and N2. What can you say about the parent pointers in the children of the original node N?
2. Suggest two ways of dealing with the inconsistent parent pointers in the children of node N.
3. For each of the above suggestions, identify a potential (major) disadvantage.
4. What conclusions can you draw from this exercise?

Exercise 9.10 Consider the instance of the Students relation shown in Figure 9.31. Show a B+ tree of order 2 in each of these cases, assuming that duplicates are handled using overflow pages. Clearly indicate what the data entries are (i.e., do not use the '$k*$' convention).

1. A dense B+ tree index on *age* using Alternative (1) for data entries.
2. A sparse B+ tree index on *age* using Alternative (1) for data entries.
3. A dense B+ tree index on *gpa* using Alternative (2) for data entries. For the purposes of this question, assume that these tuples are stored in a sorted file in the order shown in the figure: the first tuple is in page 1, slot 1; the second tuple is in page 1, slot 2; and so on. Each page can store up to three data records. You can use $\langle$*page-id, slot*$\rangle$ to identify a tuple.

Exercise 9.11 Suppose that duplicates are handled using the approach without overflow pages discussed in Section 9.7. Describe an algorithm to search for the left-most occurrence of a data entry with search key value K.

Exercise 9.12 Answer Exercise 9.10 assuming that duplicates are handled without using overflow pages, using the alternative approach suggested in Section 9.7.

sid	*name*	*login*	*age*	*gpa*
53831	Madayan	madayan@music	11	1.8
53832	Guldu	guldu@music	12	3.8
53666	Jones	jones@cs	18	3.4
53901	Jones	jones@toy	18	3.4
53902	Jones	jones@physics	18	3.4
53903	Jones	jones@english	18	3.4
53904	Jones	jones@genetics	18	3.4
53905	Jones	jones@astro	18	3.4
53906	Jones	jones@chem	18	3.4
53902	Jones	jones@sanitation	18	3.8
53688	Smith	smith@ee	19	3.2
53650	Smith	smith@math	19	3.8
54001	Smith	smith@ee	19	3.5
54005	Smith	smith@cs	19	3.8
54009	Smith	smith@astro	19	2.2

Figure 9.31 An Instance of the Students Relation

PROJECT-BASED EXERCISES

Exercise 9.13 Compare the public interfaces for heap files, B+ tree indexes, and linear hashed indexes. What are the similarities and differences? Explain why these similarities and differences exist.

Exercise 9.14 This exercise involves using Minibase to explore the earlier (non-project) exercises further.

1. Create the trees shown in earlier exercises and visualize them using the B+ tree visualizer in Minibase.
2. Verify your answers to exercises that require insertion and deletion of data entries by doing the insertions and deletions in Minibase and looking at the resulting trees using the visualizer.

Exercise 9.15 (*Note to instructors: Additional details must be provided if this exercise is assigned; see Appendix B.*) Implement B+ trees on top of the lower-level code in Minibase.

BIBLIOGRAPHIC NOTES

The original version of the B+ tree was presented by Bayer and McCreight [56]. The B+ tree is described in [381] and [163]. B tree indexes for skewed data distributions are studied in [222]. The VSAM indexing structure is described in [671]. Various tree structures for supporting range queries are surveyed in [66]. An early paper on multiattribute search keys is [433].

References for concurrent access to B trees are in the bibliography for Chapter 19.